3.4 Proving Lines are Parallel

What you should learn

GOAL 1 Prove that two lines are parallel.

GOAL 2 Use properties of parallel lines to solve **real-life** problems, such as proving that prehistoric mounds are parallel in **Ex. 19**.

Why you should learn it

▼ Properties of parallel lines help you predict the paths of boats sailing into the wind, as in **Example 4**.

REAL LIFE

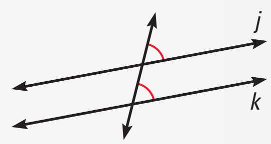
GOAL 1 Proving Lines are Parallel

To use the theorems you learned in Lesson 3.3, you must first know that two lines are parallel. You can use the following postulate and theorems to prove that two lines are parallel.

POSTULATE

POSTULATE 16 ***Corresponding Angles Converse***

If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.



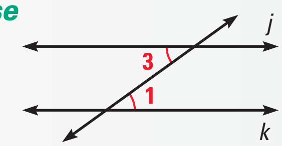
$j \parallel k$

The following theorems are converses of those in Lesson 3.3. Remember that the converse of a true conditional statement is not necessarily true. Thus, each of the following must be proved to be true. Theorems 3.8 and 3.9 are proved in Examples 1 and 2. You are asked to prove Theorem 3.10 in Exercise 30.

THEOREMS ABOUT TRANSVERSALS

THEOREM 3.8 ***Alternate Interior Angles Converse***

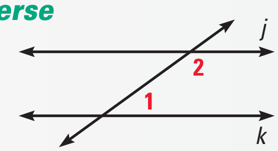
If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.



If $\angle 1 \cong \angle 3$, then $j \parallel k$.

THEOREM 3.9 ***Consecutive Interior Angles Converse***

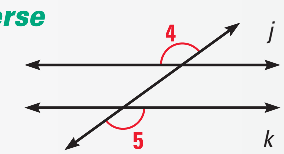
If two lines are cut by a transversal so that consecutive interior angles are supplementary, then the lines are parallel.



If $m\angle 1 + m\angle 2 = 180°$, then $j \parallel k$.

THEOREM 3.10 ***Alternate Exterior Angles Converse***

If two lines are cut by a transversal so that alternate exterior angles are congruent, then the lines are parallel.

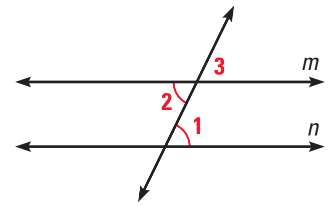


If $\angle 4 \cong \angle 5$, then $j \parallel k$.

Proof

EXAMPLE 1 *Proof of the Alternate Interior Angles Converse*

Prove the Alternate Interior Angles Converse.



SOLUTION

GIVEN ▶ $\angle 1 \cong \angle 2$

PROVE ▶ $m \parallel n$

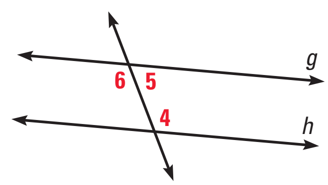
Statements	Reasons
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $\angle 2 \cong \angle 3$	**2.** Vertical Angles Theorem
3. $\angle 1 \cong \angle 3$	**3.** Transitive Property of Congruence
4. $m \parallel n$	**4.** Corresponding Angles Converse

When you prove a theorem you may use only earlier results. For example, to prove Theorem 3.9, you may use Theorem 3.8 and Postulate 16, but you may not use Theorem 3.9 itself or Theorem 3.10.

Proof

EXAMPLE 2 *Proof of the Consecutive Interior Angles Converse*

Prove the Consecutive Interior Angles Converse.



SOLUTION

GIVEN ▶ $\angle 4$ and $\angle 5$ are supplementary.

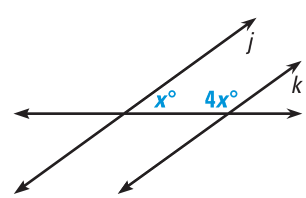
PROVE ▶ $g \parallel h$

Paragraph Proof You are given that $\angle 4$ and $\angle 5$ are supplementary. By the Linear Pair Postulate, $\angle 5$ and $\angle 6$ are also supplementary because they form a linear pair. By the Congruent Supplements Theorem, it follows that $\angle 4 \cong \angle 6$. Therefore, by the Alternate Interior Angles Converse, g and h are parallel.

Using Algebra

EXAMPLE 3 *Applying the Consecutive Interior Angles Converse*

Find the value of x that makes $j \parallel k$.



SOLUTION

Lines j and k will be parallel if the marked angles are supplementary.

$$x° + 4x° = 180°$$

$$5x = 180$$

$$x = 36$$

▶ So, if $x = 36$, then $j \parallel k$.

GOAL 2 Using the Parallel Converses

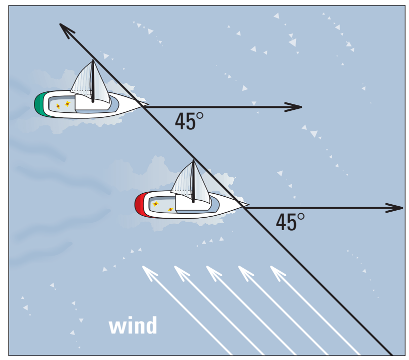
STUDENT HELP

INTERNET
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 4 *Using the Corresponding Angles Converse*

REAL LIFE

SAILING If two boats sail at a 45° angle to the wind as shown, and the wind is constant, will their paths ever cross? Explain.



SOLUTION

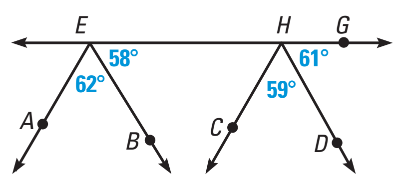
Because corresponding angles are congruent, the boats' paths are parallel. Parallel lines do not intersect, so the boats' paths will not cross.

EXAMPLE 5 *Identifying Parallel Lines*

Decide which rays are parallel.

a. Is $\overrightarrow{EB}$ parallel to $\overrightarrow{HD}$?

b. Is $\overrightarrow{EA}$ parallel to $\overrightarrow{HC}$?

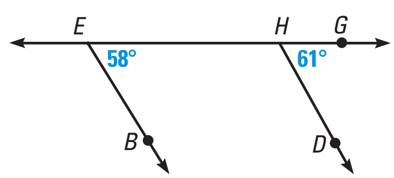


SOLUTION

a. Decide whether $\overrightarrow{EB} \parallel \overrightarrow{HD}$.

$m\angle BEH = 58°$

$m\angle DHG = 61°$



▶ $\angle BEH$ and $\angle DHG$ are corresponding angles, but they are not congruent, so $\overrightarrow{EB}$ and $\overrightarrow{HD}$ are not parallel.

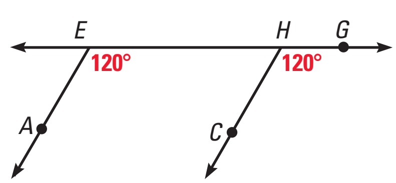
b. Decide whether $\overrightarrow{EA} \parallel \overrightarrow{HC}$.

$m\angle AEH = 62° + 58°$

$= 120°$

$m\angle CHG = 59° + 61°$

$= 120°$



▶ $\angle AEH$ and $\angle CHG$ are congruent corresponding angles, so $\overrightarrow{EA} \parallel \overrightarrow{HC}$.

GUIDED PRACTICE

Vocabulary Check ✓

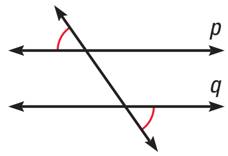
1. What are *parallel lines*?

Concept Check ✓

2. Write the converse of Theorem 3.8. Is the converse true?

Skill Check ✓

Can you prove that lines *p* and *q* are parallel? If so, describe how.

3.


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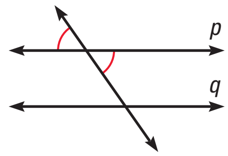

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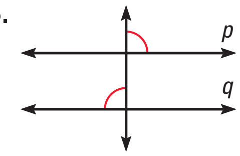

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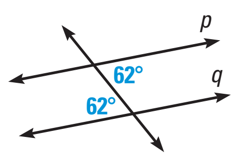

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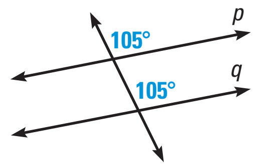

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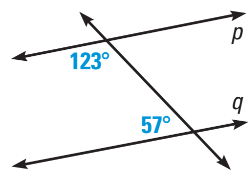
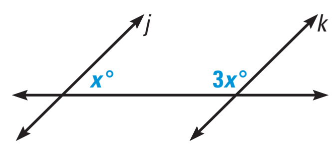


9. Find the value of x that makes $j \parallel k$. Which postulate or theorem about parallel lines supports your answer?

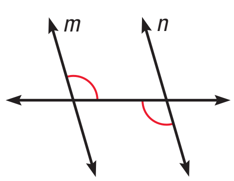


PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 808.

LOGICAL REASONING **Is it possible to prove that lines *m* and *n* are parallel? If so, state the postulate or theorem you would use.**

10.


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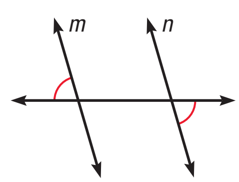

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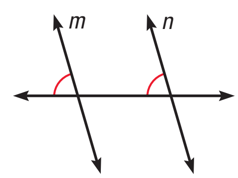

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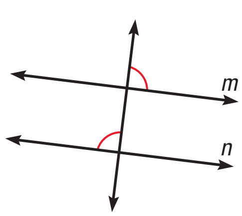

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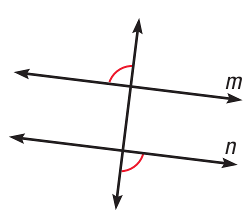

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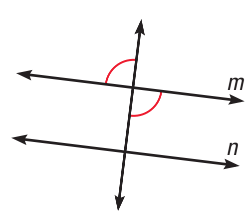
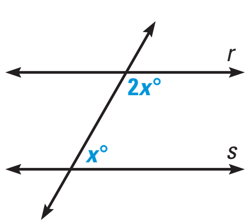


STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 28, 30
Example 2: Exs. 28, 30
Example 3: Exs. 10–18
Example 4: Exs. 19, 29, 31
Example 5: Exs. 20–27

USING ALGEBRA **Find the value of *x* that makes $r \parallel s$.**

16.


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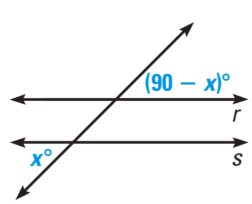

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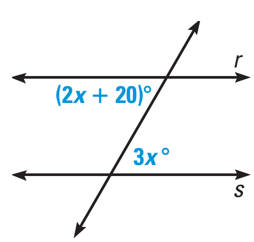
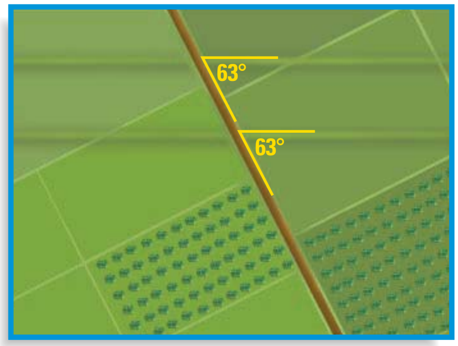


FOCUS ON APPLICATIONS

THE GREAT SERPENT MOUND, an archaeological mound near Hillsboro, Ohio, is 2 to 5 feet high, and is nearly 20 feet wide. It is over $\frac{1}{4}$ mile long.

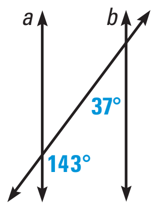
APPLICATION LINK
www.mcdougallittell.com

19. **ARCHAEOLOGY** A farm lane in Ohio crosses two long, straight earthen mounds that may have been built about 2000 years ago. The mounds are about 200 feet apart, and both form a 63° angle with the lane, as shown. Are the mounds parallel? How do you know?



LOGICAL REASONING **Is it possible to prove that lines *a* and *b* are parallel? If so, explain how.**

20.



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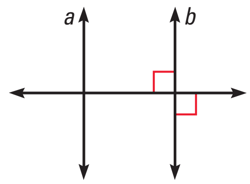

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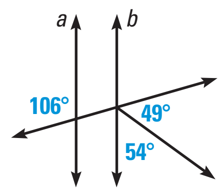

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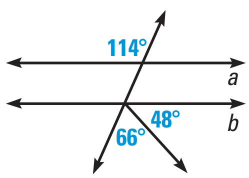

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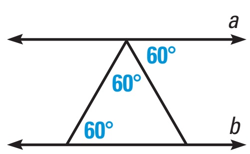

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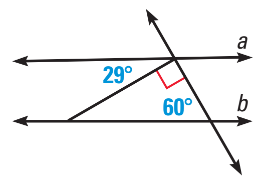
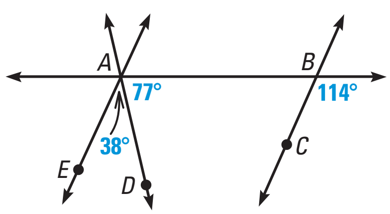
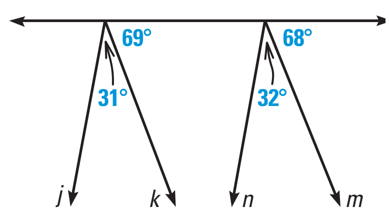


LOGICAL REASONING **Which lines, if any, are parallel? Explain.**

26.



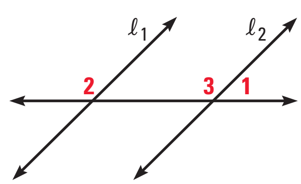
27.



28. **PROOF** Complete the proof.

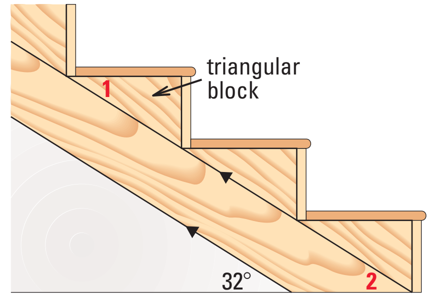
GIVEN ▶ $\angle 1$ and $\angle 2$ are supplementary.

PROVE ▶ $\ell_1 \parallel \ell_2$

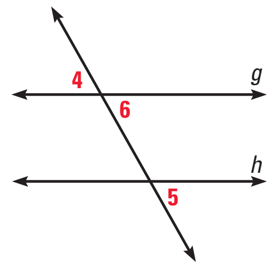


Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary.	**1.** ___?___
2. $\angle 1$ and $\angle 3$ are a linear pair.	**2.** Definition of linear pair
3. ___?___	**3.** Linear Pair Postulate
4. ___?___	**4.** Congruent Supplements Theorem
5. $\ell_1 \parallel \ell_2$	**5.** ___?___

29. **BUILDING STAIRS** One way to build stairs is to attach triangular blocks to an angled support, as shown at the right. If the support makes a 32° angle with the floor, what must $m\angle 1$ be so the step will be parallel to the floor? The sides of the angled support are parallel.



30. **PROVING THEOREM 3.10** Write a two-column proof for the Alternate Exterior Angles Converse: If two lines are cut by a transversal so that alternate exterior angles are congruent, then the lines are parallel.

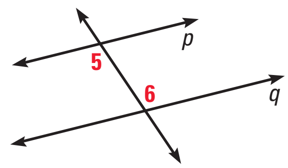


GIVEN ▶ $\angle 4 \cong \angle 5$

PROVE ▶ $g \parallel h$

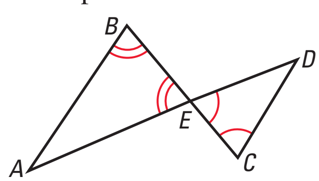
Plan for Proof Show that $\angle 4$ is congruent to $\angle 6$, show that $\angle 6$ is congruent to $\angle 5$, and then use the Corresponding Angles Converse.

31. *Writing* In the diagram at the right, $m\angle 5 = 110°$ and $m\angle 6 = 110°$. Explain why $p \parallel q$.

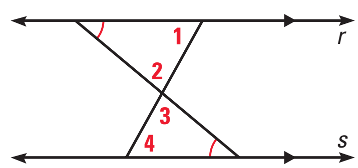


LOGICAL REASONING **Use the information given in the diagram.**

32. What can you prove about $\overline{AB}$ and $\overline{CD}$? Explain.



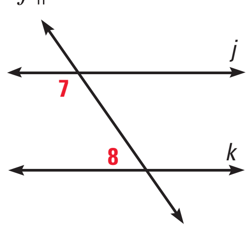
33. What can you prove about $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$? Explain.



PROOF **Write a proof.**

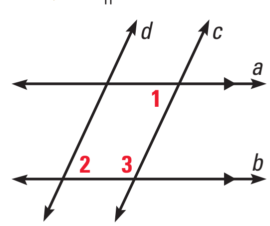
34. **GIVEN** ▶ $m\angle 7 = 125°$, $m\angle 8 = 55°$

PROVE ▶ $j \parallel k$



35. **GIVEN** ▶ $a \parallel b$, $\angle 1 \cong \angle 2$

PROVE ▶ $c \parallel d$



STUDENT HELP

SOFTWARE HELP
Visit our Web site www.mcdougallittell.com to see instructions for several software applications.

36. **TECHNOLOGY** Use geometry software to construct a line ℓ, a point P not on ℓ, and a line n through P parallel to ℓ. Construct a point Q on ℓ and construct $\overleftrightarrow{PQ}$. Choose a pair of alternate interior angles and construct their angle bisectors. Are the bisectors parallel? Make a conjecture. Write a plan for a proof of your conjecture.

Test Preparation

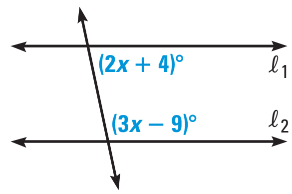
37. MULTIPLE CHOICE What is the converse of the following statement?

If $\angle 1 \cong \angle 2$, then $n \parallel m$.

Ⓐ $\angle 1 \cong \angle 2$ if and only if $n \parallel m$. Ⓑ If $\angle 2 \cong \angle 1$, then $m \parallel n$.

Ⓒ $\angle 1 \cong \angle 2$ if $n \parallel m$. Ⓓ $\angle 1 \cong \angle 2$ only if $n \parallel m$.

38. MULTIPLE CHOICE What value of x would make lines ℓ_1 and ℓ_2 parallel?



Ⓐ 13 Ⓑ 35 Ⓒ 37

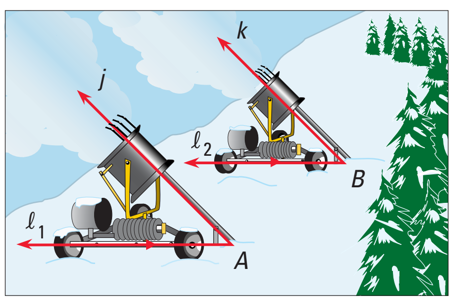
Ⓓ 78 Ⓔ 102

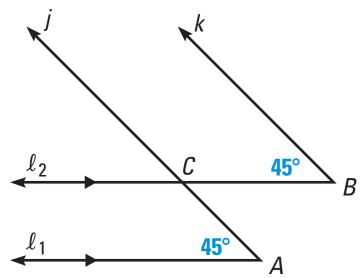
★ Challenge

39. SNOW MAKING To shoot the snow as far as possible, each snowmaker below is set at a 45° angle. The axles of the snowmakers are all parallel. It is possible to prove that the barrels of the snowmakers are also parallel, but the proof is difficult in 3 dimensions. To simplify the problem, think of the illustration as a flat image on a piece of paper. The axles and barrels are represented in the diagram on the right. Lines j and ℓ_2 intersect at C.

GIVEN ▶ $\ell_1 \parallel \ell_2$, $m\angle A = m\angle B = 45°$

PROVE ▶ $j \parallel k$





EXTRA CHALLENGE
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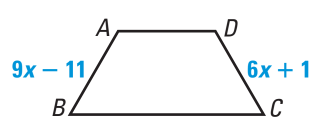
MIXED REVIEW

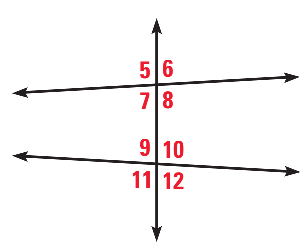
FINDING THE MIDPOINT **Use a ruler to draw a line segment with the given length. Then use a compass and straightedge to construct the midpoint of the line segment.** (Review 1.5 for 3.5)

40. 3 inches **41.** 8 centimeters **42.** 5 centimeters **43.** 1 inch

44. CONGRUENT SEGMENTS Find the value of x if $\overline{AB} \cong \overline{AD}$ and $\overline{CD} \cong \overline{AD}$. Explain your steps. (Review 2.5)



IDENTIFYING ANGLES **Use the diagram to complete the statement.** (Review 3.1)



45. $\angle 12$ and ___?___ are alternate exterior angles.

46. $\angle 10$ and ___?___ are corresponding angles.

47. $\angle 10$ and ___?___ are alternate interior angles.

48. $\angle 9$ and ___?___ are consecutive interior angles.